

Reconstructing the Cosmic Equation of State from Supernova distances

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Observations of high-redshift type Ia supernovae indicate that the universe is accelerating, fueled perhaps by a cosmological constant or by a self-interacting scalar field. In this letter, we develop a *model-independent* method for estimating the form of the scalar field potential $V(\phi)$ and the associated equation of state $w_\phi \equiv p/\varepsilon_\phi$. Our method is based on a simple yet powerful analytical form for the luminosity distance D_L which is optimized to fit observed distances to distant extragalactic supernovae, and then differentiated to yield $V(\phi)$ and w_ϕ . Our results favor $w_\phi \simeq -1$ at the present epoch, steadily increasing with redshift. However, a cosmological constant is consistent with our results. A model-independent way of obtaining the age of the universe is also proposed.

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The relation between luminosity distance and redshift for extragalactic Type Ia Supernovae (SNe) appears to favor an accelerating Universe, where almost two-thirds of the critical energy density may be in the form of a component with negative pressure [1,2]. On the other hand, several studies of large-scale structure, including those of the abundances of rich galaxy clusters [3] and clustering of galaxies [4] and Lyman- α clouds [5] (for recent reviews, see [6]) indicate low baryonic and matter densities (Ω_B, Ω_M) $\sim (0.04, 0.3)$.

This consistency is encouraging since it is well-known that a flat Cold Dark matter Universe with $\Omega_M < 1$ and a Cosmological Constant $\Lambda > 0$ fits observations of large-scale structure [6,7] better than any other theoretical model.

Although $\Lambda \neq 0$ does agree well with the recent SNe observations, it is clear that at a theoretical level a constant Λ runs into serious difficulties, since the present value of Λ is $\sim 10^{123}$ times smaller than predicted by most particle physics models [7].

A time-dependent Λ -like term, which considerably alleviates this fine-tuning problem, can be described in a simple and natural way in terms of a scalar field (referred to here as the Λ -field) with a self-interaction potential $V(\phi)$ which is minimally coupled to the Einstein gravity field, and has little or no coupling to any other known physical field. Actually, this model mimics the simplest variant of the inflationary scenario of the early Universe. Since we have not yet got a definite prediction for the form of $V(\phi)$ from theoretical considerations, it has to be reconstructed from present-day observations.

The aim of the present letter is to go from observations to theory, *i.e.* from $D_L(z)$ to $V(\phi)$, following the prescription outlined by Starobinsky [8] (see also [9]). This is the first attempt at reconstructing $V(\phi)$ from real observational data without resorting to specific models (*e.g.*

cosmological constant, quintessence etc.).

Since the spatially flat Universe ($\Omega_\phi + \Omega_M = 1$) is both predicted by the simplest inflationary models and agrees well with observational evidence, we will not consider spatially curved Friedmann-Robertson-Walker (FRW) cosmological models. In a flat FRW cosmology, the luminosity distance D_L and the coordinate distance r to an object at redshift z are simply related as ($c = 1$ here and elsewhere)

$$r \equiv a_0 \int_0^t \frac{dt}{a(t)} = \frac{D_L(z)}{1+z}. \quad (1)$$

This uniquely defines the Hubble parameter

$$H(z) \equiv \frac{\dot{a}}{a} = \left[\frac{d}{dz} \left(\frac{D_L(z)}{1+z} \right) \right]^{-1}. \quad (2)$$

For a sample of objects (in this case, extragalactic SNe Ia) for which luminosity distances D_L are measured, one can fit an analytical form to D_L as a function of z , and then estimate $H(z)$ from (2). If $\rho_m = (3H_0^2/8\pi G)\Omega_M$ is the present density of dust-like cold dark matter and the usual baryonic matter, then

$$H^2 = \frac{8}{3}\pi G \left(\rho_m + \frac{1}{2}\dot{\phi}^2 + V(\phi) \right); \text{ where} \quad (3)$$

$$\rho_m \equiv \frac{3\Omega_M H_0^2}{8\pi G} \left(\frac{a}{a_0} \right)^{-3},$$

from where it follows that

$$\dot{H} = -4\pi G(\rho_m + \dot{\phi}^2). \quad (4)$$

Eqs. (3) & (4) can be rephrased in the following form convenient for our current reconstruction exercise,

$$\frac{8\pi G}{3H_0^2} V(x) = \frac{H^2}{H_0^2} - \frac{x}{6H_0^2} \frac{dH^2}{dx} - \frac{1}{2}\Omega_M x^3, \quad (5)$$

$$\frac{8\pi G}{3H_0^2} \left(\frac{d\phi}{dx} \right)^2 = \frac{2}{3H_0^2 x} \frac{d \ln H}{dx} - \frac{\Omega_M x}{H^2}, \quad (6)$$

where $x \equiv 1 + z$. Thus from the luminosity distance D_L , both $H(z)$ and $dH(z)/dz$ can be unambiguously calculated. This allows us to reconstruct the potential $V(z)$ and $d\phi/dz$ if the value of Ω_M is additionally given. Integrating the latter equation, we can determine $\phi(z)$ (to within an additive constant) and, therefore, reconstruct the form of $V(\phi)$. Note also that the present Hubble constant $H_0 \equiv H(z=0)$ enters in a multiplicative way in all expressions. Thus, the form of the functions $H(z)/H_0$ and $V(\phi)/H_0^2$ does not depend on the actual value of H_0 .

A fitting function for D_L : We use a rational (in terms of \sqrt{x}) ansatz for the luminosity distance D_L ,

$$\frac{D_L}{x} \equiv \frac{2}{H_0} \left[\frac{x - \alpha\sqrt{x} - 1 + \alpha}{\beta x + \gamma\sqrt{x} + 2 - \alpha - \beta - \gamma} \right] \quad (7)$$

where α , β and γ are fitting parameters and $x \equiv 1 + z = a_0/a$. This function has the following important features: it is valid for a wide range of models, and it is *exactly equal* to the analytical form given by (1) for the two extreme cases: $\Omega_\phi = 0, 1$. At these two limits, as $\Omega_M \rightarrow 1$, $\alpha + \gamma \rightarrow 1$ and $\beta \rightarrow 1$; and as $\Omega_M \rightarrow 0$, $\alpha, \beta, \gamma \rightarrow 0$.

We choose this form since the value of $H(z)$ obtained by differentiating D_L/x , according to (2), has the correct asymptotic behavior: $H(z)/H_0 \rightarrow 1$ as $z \rightarrow 0$, and $H(z)/H_0 = \tilde{\Omega}_M^{1/2}(1+z)^{3/2}$ for $z \gg 1$, where

$$\tilde{\Omega}_M = \left(\frac{\beta^2}{\alpha\beta + \gamma} \right)^2. \quad (8)$$

This ensures that at high- z , the Universe has gone through a matter dominated phase. It should be noted that $\tilde{\Omega}_M$ can be larger than the CDM component Ω_M since the Λ -field (or quintessence) can have an equation of state mimicking cold matter (dust) at high redshifts. The accuracy of our ansatz is illustrated in Fig. 1.

Note that the right hand side of (6) should be non-negative for the minimally coupled scalar field model. At $z = 0$, this condition gives:

$$\frac{4\beta + 2\gamma - \alpha}{2 - \alpha} \geq 3\Omega_M, \quad (9)$$

where the equality sign occurs when the Λ -term is constant. The fact that D_L is smaller in a universe with time-dependent Λ -term than it is in a constant- Λ universe leads to a lower limit for the parameter β . When taken together with the fact that $\beta \rightarrow 1$ as $\Omega_M \rightarrow 1$ ($\Omega_\phi \rightarrow 0$) this leads to the following set of constraints

$$1 \leq \frac{1}{\beta} \leq \int_1^\infty \frac{dx}{\sqrt{1 - \Omega_M + \Omega_M x^3}}. \quad (10)$$

The observational data: Till date, about 100 SNe of Type Ia in the redshift range $z = 0.1 - 1$ have been discovered, a large fraction of which have reliable published

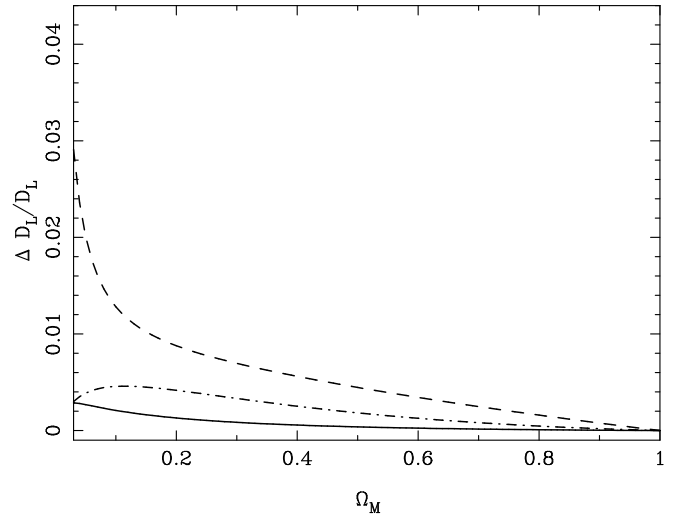


FIG. 1. The maximum deviation $\Delta D_L/D_L$ between the actual value and that calculated from the ansatz (7) in the redshift range $z=0-10$, as a function of $\Omega_M \equiv 1 - \Omega_\phi$. The three curves plotted are for constant values of the equation of state parameter (as defined in Eq. 12) $w_\phi = -1$ (solid line), $-2/3$ (dot-dashed line) and $-1/3$ (dashed line).

data from which luminosity distances can be calculated. We use the 54 SNe Ia from the preferred “primary fit” of the Supernova Cosmology Project [1], including the low- z Calan Tololo sample [10] as used in the former. To this, we add 8 supernovae from the sample of the “high- z Supernova search team” [2]. Of their ten SNe, we adopt the mean distance modulus μ_0 as calculated by their two (MLCS and template-fitting) methods, and add the two quoted errors in quadrature. We discard SN 1997ck since there is no spectroscopic evidence of its being a Type Ia SN, and SN 1996I because the two predicted distances are rather dissimilar.

TABLE I. Best-fit parameters

$\Omega_M =$	0.2	0.25	0.3	^a
$\alpha =$	$1.429^{+0.275}_{-0.615}$	$1.320^{+0.296}_{-0.645}$	$1.159^{+0.325}_{-0.704}$	
$\beta =$	$1.000^{+0.027}_{-0.028}$	$1.000^{+0.031}_{-0.032}$	$1.000^{+0.036}_{-0.039}$	
$\gamma =$	$-1.114^{+0.060}_{-0.064}$	$-1.085^{+0.067}_{-0.073}$	$-1.042^{+0.078}_{-0.088}$	
$\langle \chi^2 \rangle =$	0.91	0.91	0.92	^b
$H_0 =$	$61.6^{+1.8}_{-1.4}$	$61.4^{+1.8}_{-1.5}$	$61.2^{+1.8}_{-1.6}$	^c

^aUsed in constraint (10)

^b $\langle \chi^2 \rangle = \chi^2_{\min}/(N - 6)$, since the fit of four variables is subject to two constraints. Here $N = 62$.

^cin $\text{km s}^{-1} \text{Mpc}^{-1}$, assuming $M_0 = -19.5$.

We adopt the redshifts quoted in the above papers, and reduce them to their values in the CMB frame. This is done by adding $(30, 297, -27) \text{ km s}^{-1}$ (in Cartesian Galactic coordinates) to the heliocentric value to correct for the motion of the sun with respect to the Local Group and $(7, -542, 302) \text{ km/s}$ to correct for the motion of the LG with respect to the CMB frame. We also adopt

the errors on redshift quoted by the authors, but ensure that they are at least $\delta z = 0.002$, which is a typical peculiar velocity of a galaxy with respect to the CMB in the nearby Universe.

Maximum likelihood fits: The luminosity distance D_L (Mpc) is related to the measured quantity, the corrected apparent peak B magnitude m_B as $m_B = M_0 + 25 + 5 \log_{10} D_L$, where M_0 is the absolute peak luminosity of the SN. The function to be minimized is

$$\chi^2 \equiv \sum_{i=1}^n \frac{[y(z_i) - y(m_{Bi})]^2}{\sigma_i^2}; \quad y(z) \equiv 10^{M_0/5} D_L(z). \quad (11)$$

A fourth fitting parameter, which is required in addition to α, β, γ in the above minimization process, includes both M_0 and H_0 (which cannot be measured independent of each other). Note that this parameter only features in the fit of (7) to the data, and does not play a role in the reconstruction of $V(\phi)$.

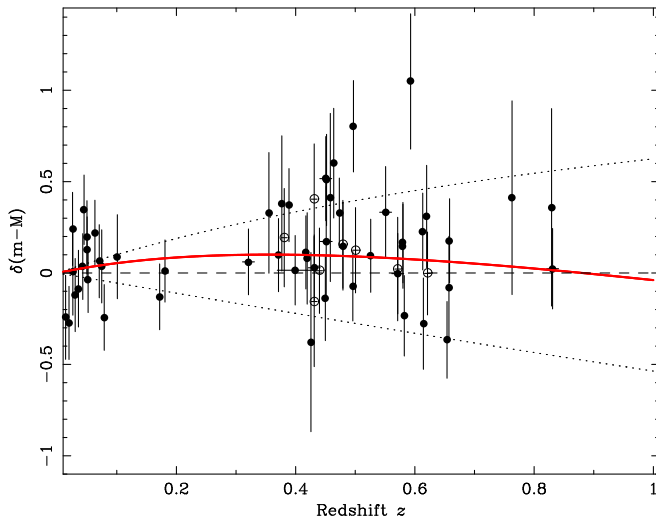


FIG. 2. The distance modulus ($m - M$) of the SNe Ia relative to an $\Omega_M \rightarrow 0$ Milne Universe (dashed line), together with the best-fit model of our ansatz (7), plotted as the solid line. The extreme cases of the $(\Omega_M, \Omega_\phi) = (0, 1)$ and $(1, 0)$ universes are plotted as dotted lines. The filled circles are the 54 SNe of [1], whereas the eight high- z SNe of [2] are plotted as open circles.

To obtain the best fit model, we perform an orthogonal chi-square fit, using errors on both the magnitude and redshift axes in σ_i , subject to the constraints (9) & (10) and $\alpha \leq 2$ to ensure that the ansatz D_L remains positive. The results are shown in Table I, and in Figure 2 (for $\Omega_M = 0.25$)

Reconstructing the scalar field potential: We show the form of the effective potential $V(z)$ reconstructed using (5) in Fig. 3, along with the corresponding plot for $V(\phi)$, where ϕ is calculated by integrating (6). The field ϕ is

determined up to an additive constant ϕ_0 , so we take ϕ to be zero at the present epoch ($z = 0$).

For a scalar field, the pressure is given by $p \equiv -T_\alpha^\alpha = \frac{1}{2}\dot{\phi}^2 - V$ and the energy density by $\varepsilon \equiv T_0^0 = \frac{1}{2}\dot{\phi}^2 + V$. The equation of state therefore is

$$w_\phi(x) \equiv \frac{p}{\varepsilon} = \frac{\frac{1}{2}\dot{\phi}^2 - V(x)}{\frac{1}{2}\dot{\phi}^2 + V(x)} = \frac{(2x/3)d \ln H/dx - 1}{1 - (H_0^2/H^2)\Omega_M x^3} \quad (12)$$

where $x = 1 + z$. For the Einstein's Cosmological constant, $w = -1$, while quintessence models [11] generally require $-1 \leq w \leq 0$ for $z \lesssim 2$.

Our experiments with several realizations of synthetic data show that this method works best if we fix the value of Ω_M . Henceforth, all reconstructed quantities are shown for $\Omega_M = 0.25$.

Our reconstruction for $w_\phi(z)$ according to (12) is plotted in in Fig. 4. We see that the condition $w_\phi \geq -1$ necessary for the model considered is satisfied in practice. There is some evidence of possible evolution in w_ϕ with $-1 \leq w_\phi \lesssim -0.8$ preferred at the present epoch, and $-1 \leq w_\phi \lesssim 0$ at $z = 1$. However, a Cosmological constant with $w = -1$ is marginally consistent with the data.

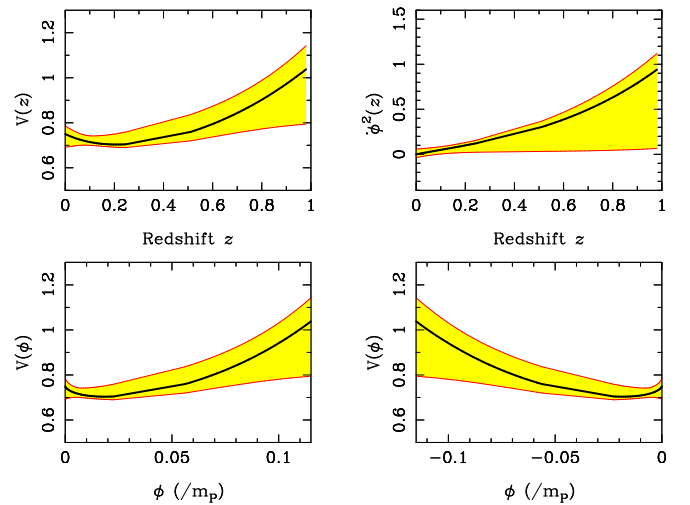


FIG. 3. The effective potential $V(z)$, and the kinetic energy term $\dot{\phi}^2$, are shown in units of $\rho_{\text{cr}} = 3H_0^2/8\pi G$. Also plotted are the two forms of $V(\phi)$ for this $V(z)$, where the errors do not reflect errors in the z - ϕ relation. The value of ϕ (known up to an additive constant) is plotted in units of the Planck mass m_P . The solid line corresponds to the best-fit values of the parameters and the shaded area covers the Monte-Carlo errors described in the text.

The errors quoted in this paper are calculated using a Monte-Carlo method, where, in a region around the best-fit values of the parameters shown in Table 1, random points are chosen in parameter space from the probability distribution function given by the χ^2 -function that is minimized to yield the best fit. At each value of z in the given range, the function in question is evaluated at over

10^7 such points, and the quoted errors enclose 68% of all the values centered on the median.

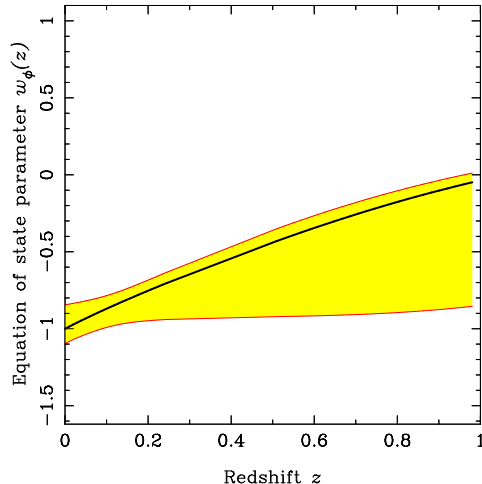


FIG. 4. The equation of state parameter $w_\phi(z) = P/\rho$ as a function of redshift. The solid line corresponds to the best-fit values of the parameters and the shaded area covers the Monte-Carlo errors as described in the text.

The ages of objects: Our ansatz (7) also provides us with a model-independent means of finding the age of the universe at a redshift z ,

$$t(z) = H_0^{-1} \int_z^\infty \frac{dz'}{(1+z')h(z')}, \quad (13)$$

where the value of $h(z) \equiv H(z)/H_0$ is determined from (2). Figure 5 shows the age of the Universe at a given z and compares it with the ages of two high redshift galaxies and the quasar B1422+231 [12]. We find that the requirement that the Universe be older than any of its constituents at a given redshift is consistent with our best-fit model, which is a positive feature since a flat matter-dominated Universe must have an uncomfortably small value of H_0 to achieve this.

Discussion: In this letter, we have proposed a simple, analytical, three parameter ansatz describing the luminosity distance as a function of redshift in a flat FRW universe. The form of this ansatz is very flexible and can be applied to determine D_L either from supernovae observations (as we have done) or from other cosmological tests such as weak lensing, the angular size-redshift relation etc. Using the resulting form of D_L we reconstruct the potential of the Λ -field (or quintessence) and the corresponding equation of state $w_\phi(z)$. Even with the limited high- z data currently available, our ansatz gives interesting results both for the form of $V(\phi)$ as well as $w_\phi(z)$. As data improve, our reconstruction promises to recover ‘true’ model independent values of $V(\phi)$ and $w_\phi(z)$ with unprecedented accuracy, thereby providing us with a deep insight into the nature of dark matter driving the acceleration of the universe.

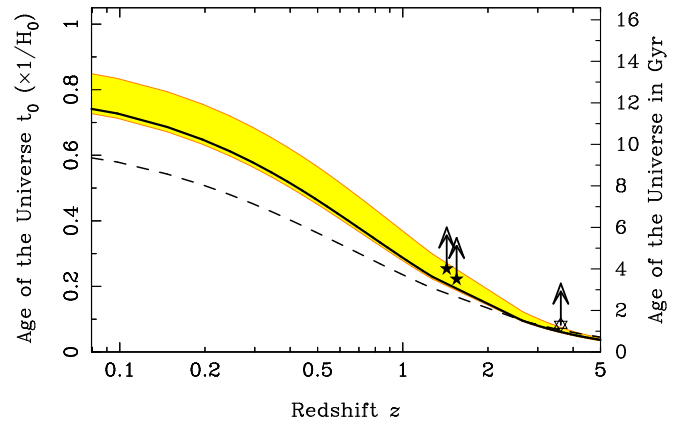


FIG. 5. The age of the Universe at a redshift z , given in units of H_0^{-1} (left vertical axis) and in Gyr, for the value of $H_0 = 61 \text{ km s}^{-1} \text{ Mpc}^{-1}$ as found in our best-fit model (right vertical axis). The shaded region represents Monte-Carlo errors as described in the text. The three high-redshift objects for which age-dating has been published [12] are plotted as lower limits to the age of the Universe at the corresponding redshifts. The dashed curve shows the same relation for an $(\Omega_M, \Omega_\phi) = (1, 0)$ Universe for the same H_0 .

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- [1] S. Perlmutter *et al.*, *Astroph. J.*, **517**, 565 (1999).
- [2] A. Riess *et al.*, *Astron. J.*, **116**, 1009 (1998).
- [3] N. A. Bahcall, X. Fan, & R. Cen, *Astroph. J.*, **485**, L53 (1997)
- [4] G. Efstathiou, W. Sutherland, & S. Maddox, *Nature*, **348**, 750 (1990)
- [5] R. A. C. Croft *et al.*, *Astroph. J.*, **520**, 1 (1999)
- [6] N. A. Bahcall, J. P. Ostriker, S. Perlmutter, & P. J. Steinhardt, *Science*, **284**, 1481 (1999)
- [7] V. Sahni, & A. A. Starobinsky, *IJMP*, to appear (1999); also astro-ph/9904398
- [8] A. A. Starobinsky, *JETP Lett.*, **68**, 757 (1998).
- [9] D. Huterer, & M. S. Turner, *Phys. Rev. D*, to appear (1999), astro-ph/9808133; T. Nakamura, & T. Chiba, *Mon. Not. Roy. ast. Soc.*, **306**, 696 (1999).
- [10] M. Hamuy *et al.*, *Astron. J.*, **112**, 2391 (1996).
- [11] R. R. Caldwell, R. Dave, & P. J. Steinhardt, *Phys. Rev. Lett.*, **80**, 1582 (1998); L. Wang, R. R. Caldwell, J. P. Ostriker, & P. J. Steinhardt, (1998) astro-ph/9901388.
- [12] J. Dunlop *et al.*, *Nature*, **381**, 581 (1996); J. Dunlop *et al.* (1998), astro-ph/9801114; Y. Yoshii, T. Tsujimoto, & K. Kawara, *Astroph. J.*, **507**, L113 (1998)